ANALYSIS OF THE HEAT TRANSFER DURING VAPOR CONDENSATION ON FREE JETS OF COLD LIQUID

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A theoretical and experimental study is shown concerning the heat transfer during vapor condensation on a flat or an axially symmetric gravity jet of cold liquid through a tap.

Vapor condensation on the surface of a free gravity jet of cold liquid is accompanied by a high rate of heat transfer, which should be of interest to the petrochemical industry in such areas as, for instance, the protection of and the cryogenic-liquids storage in low-temperature thermostatic reservoirs as well as the design of rectification columns, jet devices, and small heat exchangers. Vapor condensation on the surface of a free gravity jet of cold liquid has been studied both theoretically [1] and experimentally [2-5]. The theoretical solution to the problem is inaccurate, however, which goes back to the method of analysis. Test data are available for water only, and even these data are scarce.

This article presents a theoretical analysis of vapor condensation on a gravity jet of cold liquid and contains the results of an experimental evaluation of the heat transfer coefficient for Freon-12 vapor condensing on a jet of cold liquid Freon-12, which are important for specific design calculations but also for verifying the universality of the theoretical model.

1. Theoretical Model of the Heat Transfer Process during Vapor Condensation on a Jet of Cold Liquid

We consider a gravity jet of cold liquid discharging along the x-axis into a space filled with the same substance; the thermophysical properties of the vapor will be assumed known. Let the rate of heat transfer be governed by the rate of heat removal from the condensation surface. Let the characteristic linear dimension L in this problem be half the orifice width or the orifice radius in the flat or in the axially symmetrical case respectively, the discharge velocity at the tap be U_0, and the velocity at section x–x be U. When analyzing the problem in the hydraulic approximation, according to [1], one may in the equation of jet flow omit the terms which represent the effect of surface tension, the variation of vapor pressure along the x-axis, and the effect of precipitating condensate on the jet width or radius. Here the jet velocity is determined independently of the solution to the "thermal" problem. Assuming, for simplicity, an ideal nozzle (flow coefficient $\varphi = 1$) yields

$$u(\xi) = \sqrt{1+\alpha\xi}. \qquad (1)$$

The location of the phase-transition boundary $y = \delta(x)$ is determined from the condition of continuity with regard to the mass flow, namely in dimensionless form

$$\delta^{k+1}(\xi)\,u(\xi) = 1, \qquad (2)$$

where k = 0 corresponds to a flat jet and k = 1 corresponds to an axially symmetric jet.

In considering the "thermal" problem, we make the following assumptions:

a) the convective heat transfer is determined largely by the velocity distribution in the liquid, which in the hydraulic approximation is uniform over a transverse jet section, and

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b) the rates of molecular and turbulent heat transfer are much higher in the transverse direction than in the longitudinal direction.

Assuming, furthermore, that certain symmetry conditions of the problem are satisfied, we have on the basis of all this the following equation of heat transfer:

$$U(x)\frac{\partial T}{\partial x} = \frac{\lambda + \lambda_T(x)}{C_p\rho}\left(\frac{\partial^2 T}{\partial y^2} + \frac{k}{y}\cdot\frac{\partial T}{\partial y}\right).$$

In the case of very small velocity gradients, it may be surmised that the mechanism of turbulent heat transfer depends not on the velocity gradient but on the absolute velocity of the liquid, while the rate of molecular heat transfer is proportional to the stream velocity and to the jet width or radius [1]. In the case of a flat jet, evidently, it would be more appropriate to assume that the rate of turbulent heat transfer is proportional to the stream velocity and to the characteristic linear dimension L. In terms of similarity theory, the coefficient of turbulent heat transfer can be expressed as

$$\lambda_T(x) = \varepsilon C_p\rho U(x)\left[k\hat{\delta}(\xi) + (1-k)L\right], \tag{3}$$

where $\varepsilon = 5\cdot 10^{-4}$-$10^{-3}$ is an empirical constant based on this theory. For the majority of practical heat carrier substances, except liquid metals, $\lambda/\lambda_T \ll 1$ at discharge velocities $U_0 > 1$ m/sec and $L > 10^{-3}$ m, which allows us to disregard the molecular component of the thermal flux in the heat transfer equation. Changing now to the dimensionless variables $\eta = y/L$ and $\theta = (T_S - T)/(T_S - T_0)$, we have

$$\frac{\partial\theta}{\partial\xi} = \varepsilon\left[k\delta(\xi) + 1 - k\right]\left(\frac{\partial^2\theta}{\partial\eta^2} + \frac{k}{\eta}\cdot\frac{\partial\theta}{\partial\eta}\right). \tag{4}$$

The dimensionless temperature distribution in the jet $\theta(\xi, \eta)$ is constrained by the following system of initial and boundary conditions:

$$\theta(0, \eta) = 1; \quad \theta(\xi, \delta(\xi)) = 0; \quad \frac{\partial\theta}{\partial\eta}(\xi, 0) = 0 \tag{5}$$

The last equation in (5) is a consequence of the problem symmetry. The system of boundary conditions (5) is not convenient for use, because the unknown function is defined here on a "moving" boundary $\eta = \delta(\xi)$ [6]. It is worthwhile to change to new dimensionless variables $t \equiv \xi$ and $z = \eta/\delta(\xi)$, which will greatly simplify the system of boundary conditions. With the aid of the appropriate transformation pairs [6], Eq. (4) becomes

$$\frac{\partial\theta}{\partial t} - \frac{\delta'(t)}{\delta(t)} z\frac{\partial\theta}{\partial z} = \frac{\varepsilon\left[k\delta(t) + 1 - k\right]}{\delta^2(t)}\left(\frac{\partial^2\theta}{\partial z^2} + \frac{k}{z}\frac{\partial\theta}{\partial z}\right),$$

with $\delta'(t) = d\delta/dt$. Thus, the analysis of heat transfer according to relations (1) and (2) reduces to solving the following problem:

$$(1+\alpha\xi)\frac{\partial\theta}{\partial\xi} + \frac{\alpha z}{2(1+k)}\frac{\partial\theta}{\partial z} = \varepsilon(1+\alpha\xi)^{2-\frac{3}{4}k}\left(\frac{\partial^2\theta}{\partial z^2} + \frac{k}{z}\frac{\partial\theta}{\partial z}\right),$$

$$\theta(0, z) = 1; \quad \theta(\xi, 1) = 0; \quad \frac{\partial\theta}{\partial z}(\xi, 0) = 0. \tag{6}$$

A return to variable ξ cannot lead to ambiguities here.

It must be noted that in the equation in [1], which is analogous to Eq. (6), the term $\alpha z/2(1+k)(\partial\theta/\partial z)$ is missing. The solution obtained in [1] must not be treated as the zeroth approximation for small values of the dynamic parameter α, however, because then α would have to be made to approach zero in all coefficients of the equation without exception.

Problem (6) for small values of the dynamic parameter ($\alpha < 1$) represents the case of so-called regular perturbations [7]. Expanding the coefficients in Eq. (6) into series of positive powers in α, and expressing the unknown function in the problem as

$$\theta = \sum_{n=0}^{\infty}\alpha^n\theta_n(\xi, z), \tag{7}$$

we obtain a recurrent system of linear problems in heat conduction theory, the solution to which is rather well known. Omitting all the intermediate steps, we show here the final expressions for the local coefficient of heat transfer from vapor to liquid – accurate down to the first-degree α-terms inclusively.

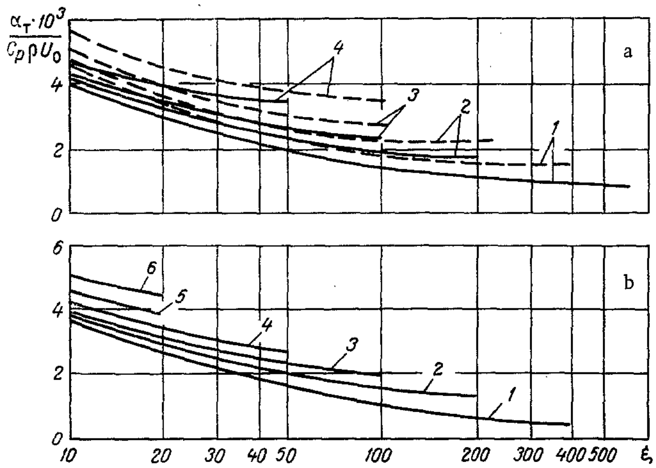

Fig. 1. Coefficient $\alpha_T / C_p \rho U_0$ as a function of the dimensionless jet length. a) Flat jet: 1) $\alpha = 0.001$; 2) 0.005; 3) 0.01; 4) 0.02. b) Axially symmetric jet: 1) $\alpha = 0.001$; 2) 0.01; 3) 0.02; 4) 0.03; 5) 0.05; 6) 0.07. Solid lines represent results obtained by these authors, dashed lines represent results in [1].

In a flat jet

$$\frac{\alpha_T}{C_p \rho U_0} = 2\varepsilon(1+\alpha\xi)\sum_{m=0}^{\infty}\exp(-\varepsilon\omega_m^2\xi) + \alpha\varepsilon\sum_{m=0}^{\infty}\exp(-\varepsilon\omega_m^2\xi)\left(\frac{\xi}{2}-\varepsilon\xi^2\omega_m^2\right) + 2\alpha\sum_{m,\,n=0}^{\infty}{}'\,\frac{\omega_m^2\left[\exp(-\varepsilon\omega_n^2\xi)-\exp(-\varepsilon\omega_m^2\xi)\right]}{(\omega_m^2-\omega_n^2)^2}, \tag{8}$$

where $\omega_m = (2m + 1)/2$ and the prime sign after the double-summation sign indicates that terms with $m = n$ are to be omitted.

In an axially symmetric jet

$$\frac{\alpha_T}{C_p \rho U_0} = \varepsilon\sum_{n=1}^{\infty}\left[2+\frac{3}{2}\alpha\xi-\frac{\alpha\varepsilon\xi^2\mu_n^2}{4}\right]\exp(-\varepsilon\mu_n^2\xi) + \alpha\sum_{k,n=1}^{\infty}{}'\,\frac{\exp(-\varepsilon\mu_k^2\xi)-\exp(\varepsilon\mu_n^2\xi)}{\mu_n^2-\mu_k^2}\cdot\frac{\mu_n}{J_1(\mu_n)J_1(\mu_k)}\int_0^1 J_0(\mu_n x)J_1(\mu_k x)x^2dx, \tag{9}$$

where μ_k are the roots of the transcendental equation $J_0(\mu_k) = 0$ and the prime sign after the double-summation sign has the same meaning as in Eq. (8).

We note that the local heat transfer coefficient α_T was defined on the basis of the characteristic in this problem temperature drop $T_S - T_0$. Using the local mean (over a transverse section of the jet) temperature drop [1] is, evidently, less convenient for practical calculations, because it results in more unwieldy formulas.

Calculated values of the local coefficient of heat transfer between vapor and liquid are shown in Fig. 1 as a function of the length coordinate and of the dynamic parameter in the problem. The dashed lines here represent values of this coefficient according to the formulas in [1] for a flat jet.

We note, in conclusion, that the solution obtained here for a flat and an axially symmetric jet can be treated as the zeroth approximation in determining how the flow dynamics and the heat transfer during vapor condensation on a free gravity jet of cold liquid are affected by the precipitating mass of condensate.

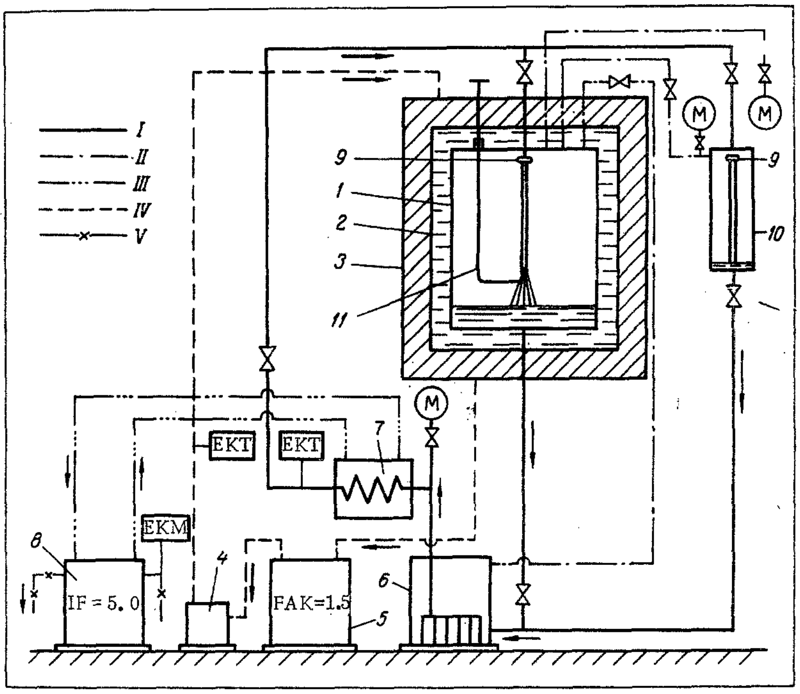

Fig. 2. Schematic diagram of the test apparatus: 1) container; 2) jacket; 3) thermal insulation; 4) gear pump; 5) model FAK-1.5 refrigerator; 6) centrifugal pump; 7) heat exchanger (evaporator); 8) model IF-50 refrigerator; 9) tap; 10) flow meter; 11) x–y plotter; I) liquid Freon-12; II) Freon-12 vapor; III) Freon-22; IV) solution of calcium chloride salt; V) water. EKT are electric-contact thermometers; EKM are electric-contact manometers; M are standard manometers.

2. Experimental Study of the Heat Transfer Process during Vapor Condensation on a Jet of Cold Liquid

We will describe here the test apparatus (Fig. 2) and the results of measurements by which the coefficient of heat transfer between vapor and liquid was determined for Freon-12 condensing on flat or axially symmetric jets of cold liquid during their medium-velocity gravity discharge through a tap into a space filled with vapor. Quantitative characteristics of this process have been established on this basis, which should be useful for engineering designs with Freon-12 as the refrigerant and also for ascertaining the reliability of the theoretical model constructed in the preceding chapter. Freon-12 vapor was condensed in a hermetic container 1 with viewing windows for visual observations and for flashlight photography. The bottom of container 1 was covered with a layer of liquid Freon-12 at a temperature within −14 to −30°C. The saturated-vapor pressure was varied here from 1.024 to 2.000 atm. abs. Container 1 was placed inside a glass jacket 2 with thermal insulation on top, through which a solution of calcium chloride salt $CaCl_2$ circulated driven by a gear pump 4 for cooling the refrigerator 5. This arrangement ensured a constant given thermal flux to the inner container 1.

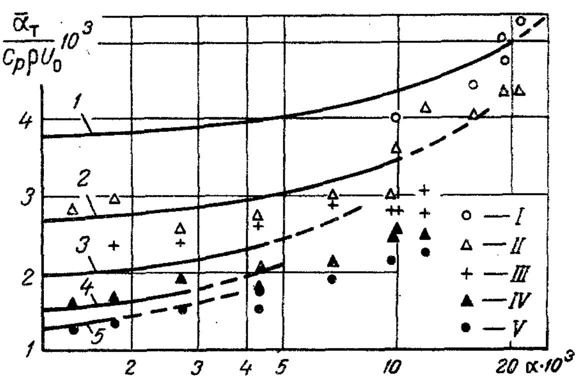

Fig. 3. Coefficient $\bar{\alpha}_T/C_p\rho U_0$ as a function of the dynamic parameter α, for a flat jet: 1) theoretical values with $\xi = 50$; 2) 100; 3) 200; 4) 400; 5) 600; I) test values with $\xi = 50$; II) 100; III) 200; IV) 400; V) 600.

In order to maintain the steady-state mode of operation, part of the liquid Freon-12 was driven by pump 6 from container 1 to the heat exchanger (evaporator) 7 of the refrigerating complex 8 and, in this way, the liquid Freon-12 was subcooled by 5-10°C. A constant temperature of the liquid at the exit from the heat exchanger was ensured by operating the refrigerator 8

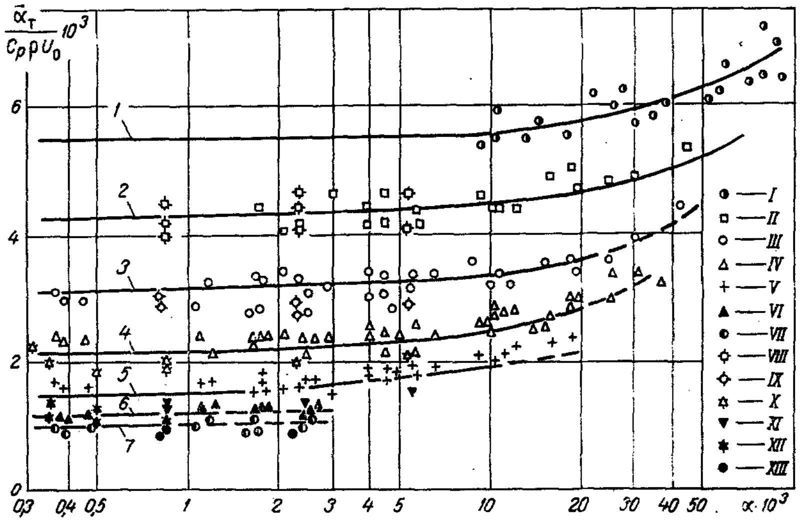

Fig. 4. Coefficient $\bar{\alpha}_T/C_p \rho U_0$ as a function of the dynamic parameter α, for an axially symmetric jet: 1) theoretical values with $\xi = 14.3$; 2) 25; 3) 50; 4) 100; 5) 200; 6) 300; 7) 400; I) test values obtained by these authors for Freon-12 with $\xi = 14.3$; II) 25; III) 50; IV) 100; V) 200; VI) 300; VII) 400; VIII) test values obtained by V. P. Isachenko for water with $\xi = 25$; IX) 50; X) 100; XI) 200; XII) 300; XIII) 400.

in the given mode, checked with an electric-contact thermometer. Vapor in container 1 condensed on the surface of the jet of subcooled Freon-12 discharging into that vapor space through the special tap 9. Stability of the processes inside container 1 was indicated by steady readings of the standard manometers and by the rhythmicity of refrigerators 5 and 8 operation. The flow rate and thus also the discharge velocity of a jet from the tap was measured by a volumetric flow meter 10 in the hydraulic system in parallel with the main container. The vapor temperature along the container height was measured with 12 thermocouples. The jet temperature at the tap exit section was measured with a thermocouple installed in the orifice and the jet temperature at various heights, at sections x = 50, 100, 200, 300, and 400 mm above the tap orifice was measured with a thermocouple which was also connected to an x–y plotter 11. From the plot we then determined the variation of the jet diameter along its height, and this made it possible to establish the heat transfer surface between jet and vapor. The thermocouples were made of copper and constantan wires 0.3 mm in diameter and calibrated.

The test stand was furnished with automatic switching for the refrigerators (including electric-contact thermometers and manometers, also a pressure relay) as well as with shutoff, regulating, and protective equipment.

Condensation of Freon-12 vapors on flat jets was studied with taps having an orifice cross section of 1 × 25 mm and 2 × 25 mm, respectively, the jet velocity at the tap orifice was varied from 0.66 to 2.8 m/sec; axially symmetric jets were studied with taps 2–7 mm in diameter at a discharge velocity from 0.5 to 7 m/sec. The maximum jet length was 0.4 m.

The coefficient of heat transfer from vapor to jet was determined on the basis of test data, according to the heat balance equation

$$\bar{\alpha}_T F_x (T_s - T_0) = \rho U_0 S (i_x - i_0). \tag{10}$$

Here F_x was determined in tests, considering the heightwise variation of the jet section. The value of coefficient $\bar{\alpha}_T$ determined according to (10) represents a mean-over-the-length value for the interval 0 to x. This coefficient is related to the local heat transfer coefficient in the preceding chapter α_T:

$$\bar{\alpha}_T = \frac{1}{x} \int_0^x \alpha_T(\varphi)\, d\varphi.$$

In Figs. 3 and 4 are shown results of experimental and theoretical studies concerning the dimensionless coefficient of heat transfer between Freon-12 vapor and liquid $\bar{\alpha}_T/C_p \rho U_0$, as a function of the dynamic parameter α and the dimensionless jet length ξ, for a flat and an axially symmetric jet, respectively. Alongside, in Fig. 4, are also shown data obtained by Isachenko for water [5].

The effect of jet splashing was not taken into account in the evaluation of test data.

Thus, this evaluation of test data pertaining to the condensation of Freon-12 and water vapors [5] on liquid jets of the same respective substance has confirmed the validity of the theoretical model of the heat transfer process within the test range of values of the governing parameters. An agreement between calculated theoretical relations and experimentally established relations, accurate within 15-20%, is feasible with only the zeroth-degree α-terms retained in expressions (8) and (9).

NOTATION

x, y	are the longitudinal and transverse coordinates of any point in the jet;
U_0	is the jet discharge velocity;
U	is the jet discharge velocity at section x–x;
$u(\xi) = U/U_0$	is the dimensionless jet discharge velocity at section $\xi-\xi$;
$\xi = x/L$	is the dimensionless longitudinal coordinate;
$\eta = y/L$	is the dimensionless transverse coordinate;
$\hat{\delta}(x)$	is the half-width or radius of the jet at section x–x;
$\delta(\xi) = \hat{\delta}(x)/L$	is the dimensionless half-width or radius of the jet at section $\xi-\xi$;
$\alpha = 2gL/U_0^2$	is the dynamic parameter in the problem;
C_p	is the specific heat of the liquid;
ρ	is the density of the liquid;
λ, λ_T	are the molecular and turbulent thermal conductivity of liquid, respectively;
$T = T(x, y)$	is the local temperature in the jet;
T_0, T_S	are the temperatures of liquid at the tap orifice and at the phase-transition boundary, respectively;
θ	is the dimensionless jet temperature;
J_0, J_1	are the Bessel functions of the first kind, of zeroth order and of first order, respectively;
F_X	is the lateral surface area of jet of length x;
S	is the area of tap orifice;
h_0, h_X	are the enthalpies of liquid at the tap orifice and at section x–x, respectively;
α_T, $\bar{\alpha}_T$	are the local and mean-over-the-length coefficient of heat transfer between vapor and liquid;
g	is the acceleration of gravity.

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